Topic and Goal:
Use frequency analysis of historical data to forecast hydrologic events

**Specific Objectives**

Be able to:
- Apply graphical frequency analysis for Log-Normal Distributions
“Recipe” for Log Normal Analysis

1. Assume the RV has log-normal distribution: create a series that consists of the logarithms $Y_i$ of the RV (e.g., annual maximum flood series)

2. Compute the sample mean (of log values) $\bar{Y}$, and standard deviation, $S_Y$

3. Define the frequency curve plotting points:
   - Point 1 = $\bar{Y} - S$ @ exceedance probability = 0.8413
   - Point 2 = $\bar{Y} + S$ @ exceedance probability = 0.1587

4. Note that 0.8413 and 0.1587 represent the probabilities that the an observation is 1 SD away from the mean of a standard normal distribution, e.g., $P(z < or = 1) = 0.8413$, and the area to the right of 1 SD = $P(z > 1) = 1 - 0.8413 = 0.1587$. But, keep in mind these are log(RV) data.

5. Check fit of data to the frequency curve: plot the data!
Log-Normal Frequency Analysis - Exercise
Given the time series of 9 annual maximum discharges shown below, develop a frequency curve assuming the log-normal distribution is applicable. Determine the probability that an annual flood peak of 42.5 m³/s will not be exceeded (non-exceedance probability in 1 year).

<table>
<thead>
<tr>
<th>Year</th>
<th>Discharge m³/s</th>
<th>Log Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>45.3</td>
<td>1.66</td>
</tr>
<tr>
<td>1992</td>
<td>27.5</td>
<td>1.44</td>
</tr>
<tr>
<td>1993</td>
<td>16.9</td>
<td>1.23</td>
</tr>
<tr>
<td>1994</td>
<td>41.1</td>
<td>1.61</td>
</tr>
<tr>
<td>1995</td>
<td>31.2</td>
<td>1.49</td>
</tr>
<tr>
<td>1996</td>
<td>19.9</td>
<td>1.30</td>
</tr>
<tr>
<td>1997</td>
<td>22.7</td>
<td>1.36</td>
</tr>
<tr>
<td>1998</td>
<td>59.0</td>
<td>1.77</td>
</tr>
<tr>
<td>1999</td>
<td>35.4</td>
<td>1.55</td>
</tr>
</tbody>
</table>
Log-Normal Frequency Analysis - Exercise

Given the time series of 9 annual maximum discharges shown below, develop a frequency curve assuming the log-normal distribution is applicable. Determine the probability that an annual flood peak of 42.5 m³/s will not be exceeded (non-exceedance probability in 1 year).

Frequency curve and the following data points plotted on probability paper; rank the data from high to low values of log Q, as follows, before computing the plotting position. Since you’re ranking high to low, use the exceedance probability (top) axis for the plotting position.

Graphical Frequency Analysis: Log-Normal Distribution

<table>
<thead>
<tr>
<th>Rank</th>
<th>log Q</th>
<th>Cunnane Plotting Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.77</td>
<td>0.0652</td>
</tr>
<tr>
<td>2</td>
<td>1.66</td>
<td>0.1739</td>
</tr>
<tr>
<td>3</td>
<td>1.61</td>
<td>0.2826</td>
</tr>
<tr>
<td>4</td>
<td>1.55</td>
<td>0.3913</td>
</tr>
<tr>
<td>5</td>
<td>1.49</td>
<td>0.5000</td>
</tr>
<tr>
<td>6</td>
<td>1.44</td>
<td>0.6087</td>
</tr>
<tr>
<td>7</td>
<td>1.36</td>
<td>0.7174</td>
</tr>
<tr>
<td>8</td>
<td>1.30</td>
<td>0.8261</td>
</tr>
<tr>
<td>9</td>
<td>1.23</td>
<td>0.9348</td>
</tr>
</tbody>
</table>

\[ P_i = \frac{i - 0.4}{n + 0.2} \]
Log-Normal Frequency Analysis - Exercise

Given the time series of 9 annual maximum discharges shown below, develop a frequency curve assuming the log-normal distribution is applicable. Determine the probability that an annual flood peak of 42.5 m$^3$/s will not be exceeded (non-exceedance probability in 1 year).

Frequency curve and the following data points plotted on probability paper; rank the data from high to low values of log Q, as follows, before computing the plotting position. Since you’re ranking high to low, use the exceedance probability (top) axis for the plotting position.

<table>
<thead>
<tr>
<th>Rank</th>
<th>log Q</th>
<th>Cunnane Plotting Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.77</td>
<td>0.0652</td>
</tr>
<tr>
<td>2</td>
<td>1.66</td>
<td>0.1739</td>
</tr>
<tr>
<td>3</td>
<td>1.61</td>
<td>0.2826</td>
</tr>
<tr>
<td>4</td>
<td>1.55</td>
<td>0.3913</td>
</tr>
<tr>
<td>5</td>
<td>1.49</td>
<td>0.5000</td>
</tr>
<tr>
<td>6</td>
<td>1.44</td>
<td>0.6087</td>
</tr>
<tr>
<td>7</td>
<td>1.36</td>
<td>0.7174</td>
</tr>
<tr>
<td>8</td>
<td>1.30</td>
<td>0.8261</td>
</tr>
<tr>
<td>9</td>
<td>1.23</td>
<td>0.9348</td>
</tr>
</tbody>
</table>
Log-Normal Frequency Analysis - Exercise
Given the time series of 9 annual maximum discharges shown below, develop a frequency curve assuming the log-normal distribution is applicable. Determine the probability that an annual flood peak of 42.5 m$^3$/s will not be exceeded (non-exceedance probability in 1 year).

Frequency curve and the following data points plotted on probability paper; rank the data from high to low values of log Q, as follows, before computing the plotting position. Since you’re ranking high to low, use the exceedance probability (top) axis for the plotting position.

<table>
<thead>
<tr>
<th>Rank</th>
<th>log Q</th>
<th>Cunnane Plotting Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.77</td>
<td>0.0652</td>
</tr>
<tr>
<td>2</td>
<td>1.66</td>
<td>0.1739</td>
</tr>
<tr>
<td>3</td>
<td>1.61</td>
<td>0.2826</td>
</tr>
<tr>
<td>4</td>
<td>1.55</td>
<td>0.3913</td>
</tr>
<tr>
<td>5</td>
<td>1.49</td>
<td>0.5000</td>
</tr>
<tr>
<td>6</td>
<td>1.44</td>
<td>0.6087</td>
</tr>
<tr>
<td>7</td>
<td>1.36</td>
<td>0.7174</td>
</tr>
<tr>
<td>8</td>
<td>1.30</td>
<td>0.8261</td>
</tr>
<tr>
<td>9</td>
<td>1.23</td>
<td>0.9348</td>
</tr>
</tbody>
</table>

Hint: probably best to set your ordinate axis (Log Q) to start at 1.0 with 0.1 increments, up to 1.9, since 1.77 is the highest value.
Graphical Approach

Log Normal Distributions

CVEEN 4410 Engineering Hydrology

Define frequency curve using the following two points:

+ $S = 1.4896 + 0.177 = 1.6666$
- $S = 1.4896 - 0.177 = 1.3126$

Point 1.
Ordinate: $\log Q = 1.3126$
Abscissa: Exceedance $P = 0.8413$

Point 2.
Ordinate: $\log Q = 1.6666$
Abscissa: Exceedance $P = 0.1587$

Graphical Frequency Analysis

Graphical Frequency Analysis: Log Normal Distribution

Point 1: $\log Q = 1.77, P = 0.0652$

Hint: probably best to set your ordinate axis ($\log Q$) to start at 1.0 with 0.1 increments, up to 1.8, since 1.77 is the highest value.

Since you’re ranking high to low, use the exceedance probability (top) axis for the plotting position.

$1-F(x)\%$

Exceedance Probability

Non-Exceedance Probability

F(x) [%]
Graphical Approach

Log Normal Distributions

CVEEN 4410 Engineering Hydrology

Define frequency curve using the following two points:

\[ S = 1.4896 + 0.177 = 1.6666 \]

\[ -S = 1.4896 - 0.177 = 1.3126 \]

Point 1.

Ordinate: \( \log Q = 1.3126 \)
Abscissa: Exceedance \( P = 0.8413 \)

Point 2.

Ordinate: \( \log Q = 1.6666 \)
Abscissa: Exceedance \( P = 0.1587 \)

Graphical Frequency Analysis

Objective:

Graphical Frequency Analysis: Log Normal Distribution

Point 1: \( \log Q = 1.77 \) \( P = 0.0652 \)
Define frequency curve using the following two points:

- $S = 1.4896 + 0.177 = 1.6666$
- $S = 1.4896 - 0.177 = 1.3126$

Point 1.
- Ordinate: $\log Q = 1.3126$
- Abscissa: Exceedance $P = 0.8413$

Point 2.
- Ordinate: $\log Q = 1.6666$
- Abscissa: Exceedance $P = 0.1587$

Graphical Frequency Analysis Objectives

Graphical Frequency Analysis: Log Normal Distribution

Point 2: $\log Q = 1.66 \quad P = 0.1739$
Define frequency curve using the following two points:

Point 1. Ordinate: log Q = 1.3126, Abscissa: Exceedance P = 0.8413

Point 2. Ordinate: log Q = 1.6666, Abscissa: Exceedance P = 0.1587

Point 3: Log Q = 1.61, P = 0.2826
Define frequency curve using the following two points:

\[ S = 1.4896 + 0.177 = 1.6666 \]

\[-S = 1.4896 - 0.177 = 1.3126 \]

Point 1.

Ordinate: \( \log Q = 1.3126 \)
Abscissa: Exceedance \( P = 0.8413 \)

Point 2.

Ordinate: \( \log Q = 1.6666 \)
Abscissa: Exceedance \( P = 0.1587 \)

Now, these data can be compared to the frequency curve corresponding to the data set (population).
Log-Normal Frequency Analysis - Exercise

Given the time series of 9 annual maximum discharges shown below, develop a frequency curve assuming the log-normal distribution is applicable. Determine the probability that an annual flood peak of 42.5 m$^3$/s will not be exceeded (non-exceedance probability in 1 year).

Compute moments:

- **Mean**: 1.4896
- **Standard deviation**: 0.177

Define frequency curve using the following two points:

- **Point 1.** Ordinate: log $Q = 1.3126$  
  Abscissa: Exceedance $P = 0.8413$
- **Point 2.** Ordinate: log $Q = 1.6666$  
  Abscissa: Exceedance $P = 0.1587$
Define frequency curve using the following two points:

- S = 1.4896 + 0.177 = 1.6666
- S = 1.4896 – 0.177 = 1.3126

Point 1. Ordinate: log Q = 1.3126 Abscissa: Exceedance P = 0.8413
Point 2. Ordinate: log Q = 1.6666 Abscissa: Exceedance P = 0.1587
Data fit curve reasonably well. Therefore, determine non-exceedance probability for 42.5 m$^3$/s. log$10$(42.5) = 1.63, which on the frequency curve corresponds to a non-exceedance probability (lower abscissa) of 0.8, or

$$F(Y < 42.5) = 0.80$$

→ Non-exceedance probability (probability that it will not be exceeded in one year!)
Next time: Data that fit Log-Pearson III Distribution