Class 17: Unit Hydrograph Convolution
Topic and Goal:
Convolve a unit hydrograph with a design storm to derive a (predicted) storm hydrograph.
Excess Rain Becomes Direct Runoff

Outflow resulting from 3.3 in. of rain over a 3.5 mi² watershed in Northwest Houston
Recall that the Unit Hydrograph is a depiction of the hydrograph (discharge) that would result from 1.0 inch of rainfall.
Unit Hydrographs and Convolution

The Unit Hydrograph may be used as a “transfer function” to convert observed storms or design storms into real-time predictions of ultimate discharge.

Such predictions are essential for effective engineering design for detention basins, drainage systems, etc.
Class 17: Unit Hydrographs

Learning Objectives

Excess Rain Becomes Direct Runoff

Purposes of unit hydrographs and convolution

Convolution Equation

In-Class Exercise

One instance where a textbook says it best:

Unit Hydrograph Convolution

The real importance of the UH approach is demonstrated in the development of storm hydrographs due to an actual rainfall event over a watershed. UH ordinates $U_j$ are multiplied by rainfall excess $P_n$ and added and lagged in a sequence to produce a resulting storm hydrograph, which ideally can be done using available Excel spreadsheets. Hydrograph convolution, the procedure of deriving a storm hydrograph from a multiperiod rainfall excess, is illustrated in Example 2–5 and represents a more general case of the time-area method [Eq. (2–2)]. For actual watersheds, these calculations are almost always done in a hydrologic model such as HEC-HMS (see Chapter 5).

Each unit hydrograph is added at the time corresponding to the rainfall spike that produced a response. Base flow can be added to produce a realistic storm hydrograph if base flow values are available for the watershed under study. Care should be taken in this calculation to ensure that the time increments of rainfall excess correspond exactly to the duration of the UH. For example, 1-hr time increments should be used with the 1-hr UH. The governing equation for the storm hydrograph in discrete form is called the convolution equation:

$$Q_n = \sum_{i=1}^{n} P_i U_{n-i+1}, \quad (2-4)$$

or

$$Q_n = P_n U_1 + P_{n-1} U_2 + P_{n-2} U_3 + \cdots + P_1 U_n,$$

where $Q_n$ is the storm hydrograph ordinate, $P_i$ is rainfall excess, and $U_j (j = n - i + 1)$ is the unit hydrograph ordinate. Periods of no rainfall can also be included, as shown in Example 2–5.
In-Class Exercise: Estimating Hydrographs via UH Convolution

Convolve a design storm (in this case, net rainfall) consisting of

\[ P_n = [0.5, 1.0, 1.5, 0.0, 0.5] \text{ inches over 5 hours, with 1-hour increments,} \]
(i.e., 0.5 inches the first hour, 1.0 the next, etc.)

with the following 1-hour unit hydrograph:

\[ U_n = [0, 100, 320, 450, 370, 250, 160, 90, 40, 0] \text{ cfs} \]

**but note:** the hydrograph units may be thought of as cfs/inch, because recall that to create the unit hydrograph, you divided the original hydrograph by the volume of DRO in inches; then, when you convolve a design storm (in inches) with a unit hydrograph (in cfs/inch), that convolution or multiplication results in the predicted hydrograph with units of cfs.